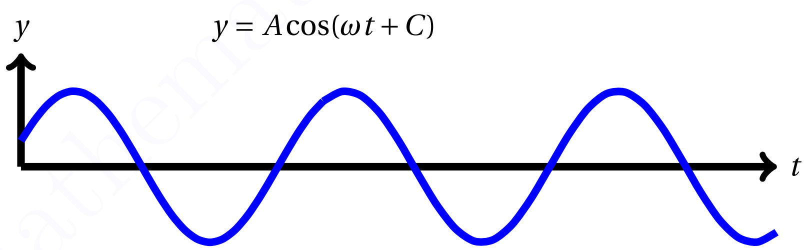
7.6: Modeling with Trigonometric Equations.

- How to Form a Sinusoidal Function: $A\cos(Bt+C)+D$ or $A\sin(Bt+C)+D$

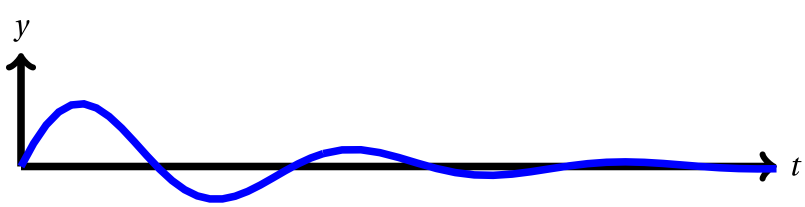
  - Use the clues to find how long it takes for events to repeat.(Period) Sometimes the number of repetition is given in 1 second, 1 minutes and etc. Then divide 2π (for Sine and Cosine) or π (for Tangent and Cotangent) by the number of repetition to find the period.
  - Remember that the period is also $\dfrac{2\pi}{|B|}$ or $\dfrac{\pi}{|B|}$. Use that to find B. Alternatively, it may be easier to use a composition of trigonometric function and an angle that is a function of time or arc function and a ratio that is function of time.
  - The average value gives the midline of the function. ($y = D$)
  - Half the difference between the max value and the min value gives the amplitude (A). The difference between any of the extremum points and average is also amplitude.
  - Use clues about time when an event happens, such as max, min and average values, to decide between sine, cosine and find a phase shift if it exists.

Simple (free) Harmonic Motions or Damped Harmonic Motions for $t > 0$.

- A free harmonic motion follows functions $f(t) = A\sin(\omega t + C)$ or $g(t) = A\cos(\omega t + C)$. This is oscillation forever.



- Damped harmonic motion follows $f(t) = Ae^{-rt}\sin(\omega t + C)$ or $g(t) = e^{-rt}A\cos(\omega t + C)$, $r > 0$. This is a dying oscillation. That is the function oscillates around the horizontal asymptote.

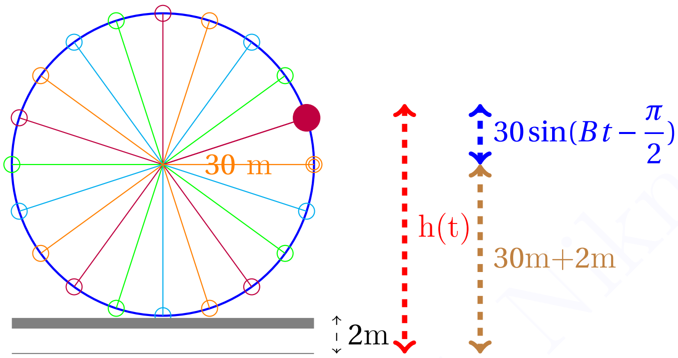


- The motion of a spring (seen earlier in Section 6.1) is a harmonic motion. In that example, damping was not considered.

We only do limited number of problems. Refer to the previous sections for more modeling problems.

1. A Ferris wheel is 60 meters in diameter and boarded from a platform that is 2 meters above the ground. The six o'clock position on the Ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 6 minutes. Baby Ava, who is riding for one rotation, giggles when the ride is higher than 17 meters above the ground and is quiet the rest of the time. How much of the ride, in minutes, is she giggling?

   (Hint: Find the equation of the height at any time t. Then compute when height is 17 first and when it is 17 next.)

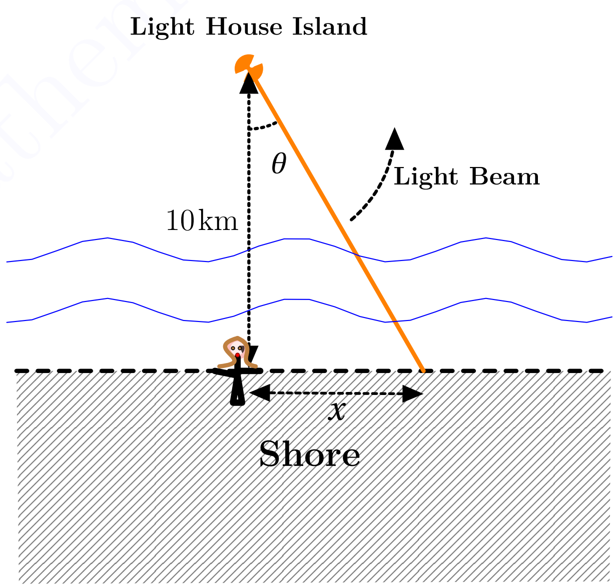


2. A lighthouse sits on a small island near a rocky shoreline, emitting a rotating beam of light. The lighthouse is $10\,km$ from the shore, and it rotates 9 times per minute, at a constant rate. Nora is standing on the shoreline where the vertical line meets the shore.

(A) Express the distance of the end of the light beam on the shore from Nora x, as a function of θ, the angle that light beam makes with the vertical line; see picture below.

(B) When the beam is making angle $\theta = 30°$ with vertical line, how far is the end of the light beam from Nora?

(C) If at time $t = 0$, the light is directly shining on Nora, express θ as a function of time, then use Part (A) to express x as a function of time t, in minutes.



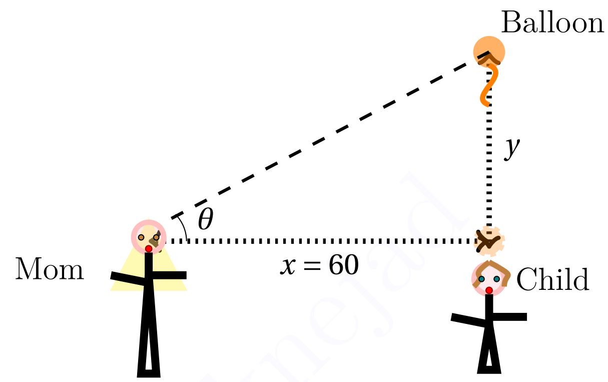
3. Outside temperatures over the course of a day can be modeled as a sinusoidal function. Suppose the temperature varies between $63°F$ and $87°$ during the day and the average daily temperature first occurs at 12 AM. Temperature drops for a while after midnight and starts rising at sunrise.

   (A) What is the midline of this function?

   (B) What is the period of the function?

   (C) How many hours after midnight does the temperature first reach $81°F$?

4. Albert lets go of his balloon and the balloon starts rising vertically at a constant speed of 15 feet per second. Mom who is standing 60 feet from Albert is looking directly at the balloon, which was in her horizontal line of sight in the beginning, stays frozen in her place and keeps looking straight at the balloon as it is rising. Let y be the vertical distance between Albert and the balloon, in feet, and θ be the angle of mom's line of sight with horizontal line.



(a) Express the distance between the mom and the balloon, D, as an expression of y.

(b) Express y as a function of time in seconds, t.

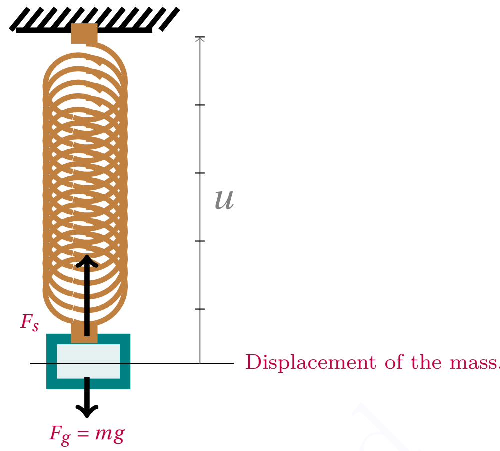
(c) Express D as a function of t.

(d) Express θ as a function of y.

(e) Express θ as a function of t.

5. **Mechanical Engineering:** Displacement of a damped spring mass system hanging from a ceiling is modeled by

$u(t) = 5e^{-t}\cos(5t) + 3$ in cm where t is in seconds.



https://www.geogebra.org/m/p9apbz27

(a) What is the displacement at time $t = 0$ seconds?

(b) What is the displacement in the long run?

(c) What is the equilibrium position for the spring?

(d) Interpret your answers in Parts (a)-(c).

Video:

1. https://mediahub.ku.edu/media/t/1_dpy11wd5
2. https://mediahub.ku.edu/media/t/1_g32e5vjk